Quantum Measurement, Wavefunction Collapse, and Entanglement on a Quantum Computer

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Representing Multiple Qubits

Let's consider a system containing two spin-up qubits:

$$|\uparrow\rangle \otimes |\uparrow\rangle$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1(1) \\ 1(0) \\ 0(1) \\ 0(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$|\uparrow\rangle \otimes |\uparrow\rangle = |\uparrow\uparrow\rangle$$

A Two Qubit System

- Options:
  - $|\uparrow\uparrow\rangle$
  - $|\uparrow\downarrow\rangle$
  - $|\downarrow\uparrow\rangle$
  - $|\downarrow\downarrow\rangle$
- Problem: Compute the vector representations of these four states.

Qiskit Code

```
# Needed to set up the quantum circuit
from qiskit import QuantumCircuit, ClassicalRegister,\
  QuantumRegister
# Create two qubits
# Map two qubits to two clasical bits
q = QuantumRegister(2)
c = ClassicalRegister(2)
qc = QuantumCircuit(q, c)
qc.measure(q, c)
```

- The result will be the state $|\uparrow\uparrow\rangle$ ($|00\rangle$).

Bell States

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Creating the Bell States

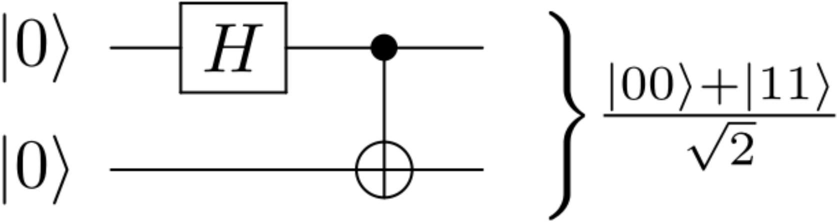


Figure 1: Circuit Diagram for the $|\Phi^+\rangle$ State

Creating the Bell States

```
q = QuantumRegister(2)
c = ClassicalRegister(2)
qc = QuantumCircuit(q, c)
# Add a H gate on qubit 0
qc.h(0)
# Add a CX (CNOT) gate on control qubit 0 and
#target qubit 1
qc.cx(0, 1)
```

Creating the Bell States

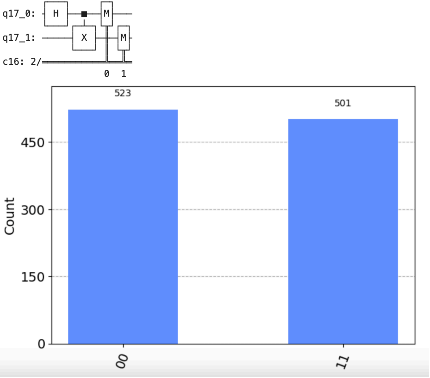


Figure 2: Qiskit Results from Creating $|\Phi^+\rangle$

Gates Involving Two Qubits

- A gate acting on n qubits is represented by a $2^n x 2^n$ unitary matrix
- Most two qubit gates are controlled gates
  - One qubit is the control qubit $\longrightarrow$ its value determines the behavior of the gate and is not changed
  - One qubit is the target qubit $\longrightarrow$ its value could be changed depending on the value of the control qubit

Gates Involving Two Qubits

- Example 1: The Controlled NOT or CNOT Gate (CX)
  - Control Qubit and Target Qubit
  - If control qubit is 0, do nothing
  - If control qubit is 1, apply a NOT gate to the target qubit
- Exercise: Create a *case table* for the CNOT gate.
- Exercise: Does this gate's use in the creation of $|\Phi^+\rangle$ make sense?

$$CX = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

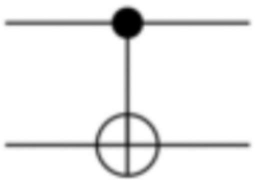


Figure 3: CX Gate Symbol

Gates Involving Two Qubits

- Example 2: The Controlled Z Gate
  - If the control qubit is $|\uparrow\rangle$ the gate does nothing
  - If the control qubit is $|\downarrow\rangle$ the gate applies a Z gate to the target qubit
- Exercise: Create a *case table* for the CZ gate.

$$CZ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

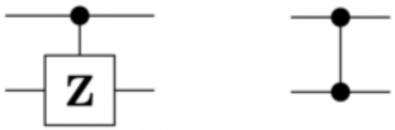


Figure 4: CZ Gate Symbol

Gates Involving Two Qubits

- Example 3: Swap Gate
  - Swaps the states of two qubits
  - Composed of three sequential CNOT gates

$$SWAP = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

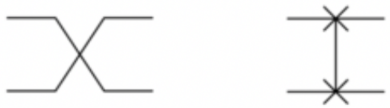
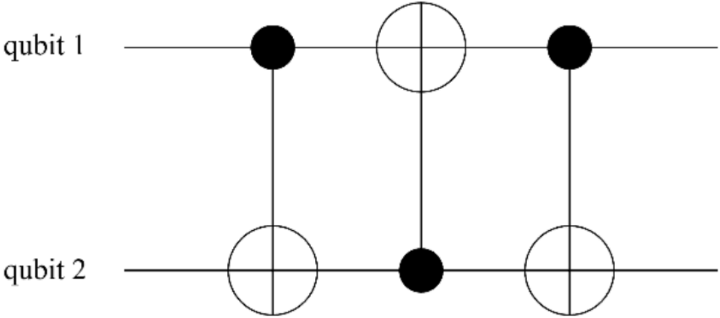


Figure 5: SWAP Gate Symbol

Gates Involving Two Qubits

- Example 3: Swap Gate



A Three Qubit System

- Ideas about notation?
- Number of three qubit combinations?
- What size and type of matrices would represent three qubit gates?

Gates Involving Three Qubits

- Example: Fredkin Gate or Controlled SWAP gate
  - If the control qubit is $|\uparrow\rangle$, do nothing to the two target qubits.
  - If the control qubit is $|\downarrow\rangle$, apply a SWAP gate to the two target qubits.

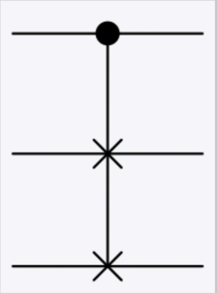


Figure 6: The Fredkin Gate

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Figure 7: The Fredkin Gate Matrix Representation