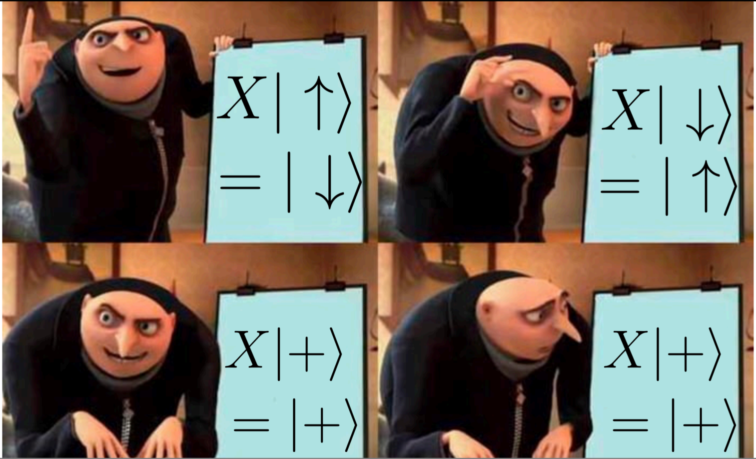
Deutsch-Jozsa Algorithm

Julie Butler

But First a Meme

[redacted]



Problem Statement

- Assume there is a black box algorithm (oracle) which accepts a binary string of length n and has one of the following behaviors:
  - Always returns 0 (constant oracle)
  - Always returns 1 (constant oracle)
  - Returns 0 for $\frac{1}{2}$ of the possible inputs and 1 for the other inputs (balanced oracle)
- **Problem:** Determine the function of the given oracle

Classical Solution

- For a binary string of length n, there are 2^n possible strings
- If you test a string and get 1, it could be constant or balanced
- If you test two strings and get 1 both times, it could be constant or balanced
- If you test three strings and get 1 all three times, it could be constant or balanced
- ...
- If you test $\frac{2^n}{2} + 1$ string and you get 1 every time then you can say it is constant

Classical Solution Continued

- To ensure you know the function of the oracle you need to test $\frac{2^n}{2} + 1$ different binary strings
  - n $=$ 1, test 2 strings
  - n $=$ 4, test 9 strings
  - n $=$ 5, test 17 strings
  - n $=$ 7, test 65 strings
  - ...
- Query method: $\frac{2^n}{2} + 1$ queries for worst case/be sure

Outlining the Quantum Solution

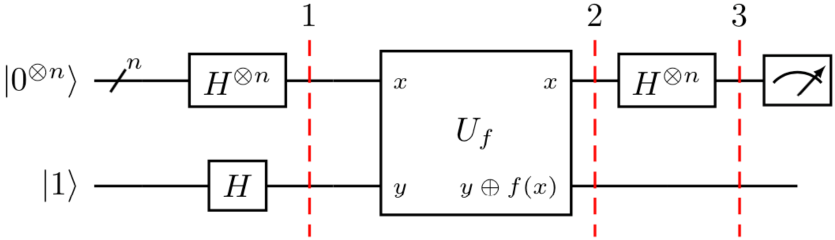


Figure 2: Outlining the Quantum Solution

- Query method: only one query of the function needed
- Note Deutsch-Jozsa's algorithm for n = 1 becomes a special case called Deutsch's algorithm

Focus only on the top qubits

- What state do we have after the Hadamard gates for:
  - $n = 1$
  - $n = 2$
  - $n = 3$
  - ...
- Quantum parallelism!

Qiskit Time!