Deutsch-Jozsa Algorithm

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Deutsch-Jozsa Algorithm

But First a Meme



Deutsch-Jozsa Algorithm

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Problem Statement



Assume there is a black box algorithm (oracle) which accepts a binary string of length n and has one of the following behaviors:

Always returns 0 (constant oracle)

- Always returns 1 (constant oracle)
- Returns 0 for $\frac{1}{2}$ of the possible inputs and 1 for the other inputs (balanced oracle)
- **Problem:** Determine the function of the given oracle

Classical Solution

- \blacktriangleright For a binary string of length n, there are 2^n possible strings
- ▶ If you test a string and get 1, it could be constant or balanced
- If you test two strings and get 1 both times, it could be constant or balanced
- If you test three strings and get 1 all three times, it could be constant or balanced

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If you test $\frac{2^n}{2} + 1$ string and you get 1 every time then you can say it is constant

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Classical Solution Continued

To ensure you know the function of the oracle you need to test ²ⁿ/₂ + 1 different binary strings

n = 1, test 2 strings
n = 4, test 9 strings
n = 5, test 17 strings
n = 7, test 65 strings
...

Query method: ²ⁿ/₂ + 1 queries for worst case/be sure

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Outlining the Quantum Solution



Figure 2: Outlining the Quantum Solution

- Query method: only one query of the function needed
- Note Deutsch-Jozsa's algorithm for n = 1 becomes a special case called Deutsch's algorithm

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Focus only on the top qubits

What state do we have after the Hadamard gates for:
n = 1
n = 2
n = 3
...
Quantum parallelism!

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Qiskit Time!

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