Example 4

$|\psi\rangle = A\begin{bmatrix} 3i \\ 4 \end{bmatrix} \Rightarrow$ based on in class example $A = 1/5$

$|\psi\rangle = \frac{1}{5}\begin{bmatrix} 3i \\ 4 \end{bmatrix}$

$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Now to compute the expectation values

$$\langle\psi|S_x|\psi\rangle = \frac{1}{5}(-3i \quad 4)\frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\frac{1}{5}\begin{pmatrix} 3i \\ 4 \end{pmatrix}$$

$$= \frac{\hbar}{50}(-3i \quad 4)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3i \\ 4 \end{pmatrix}$$

$$= \frac{\hbar}{50}(-3i \quad 4)\begin{pmatrix} 4 \\ 3i \end{pmatrix}$$

$$= \frac{\hbar}{50}(-3i(4) + 4(3i)) = 0 = \langle S_x\rangle$$

$$\langle\psi|S_y|\psi\rangle = \frac{1}{5}[-3i \quad 4]\frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\frac{1}{5}\begin{pmatrix} 3i \\ 4 \end{pmatrix}$$

$$= \frac{\hbar}{50}[-3i \quad 4]\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{bmatrix} 3i \\ 4 \end{bmatrix}$$

$$= \frac{\hbar}{50}[-3i \quad 4]\begin{bmatrix} -4i \\ -3 \end{bmatrix} = \frac{\hbar}{50}(-3i(-4i) + 4(-3))$$

$$= \frac{\hbar}{50}(-12 + -12) = \frac{-24\hbar}{50}$$

$$= \frac{-12}{25}\hbar = \langle S_y\rangle$$

$$\langle \psi | S_z | \psi \rangle = \frac{1}{5} \begin{bmatrix} -3i & 4 \end{bmatrix} \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{1}{5} \begin{bmatrix} 3i \\ 4 \end{bmatrix}$$

$$= \frac{\hbar}{50} \begin{bmatrix} -3i & 4 \end{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{bmatrix} 3i \\ 4 \end{bmatrix}$$

$$= \frac{\hbar}{50} \begin{bmatrix} -3i & 4 \end{bmatrix} \begin{bmatrix} 3i \\ -4 \end{bmatrix} = \frac{\hbar}{50} \left(-3i(3i) + 4(-4) \right)$$

$$= \frac{\hbar}{0} (9 - 16)$$

$$= \frac{7\hbar}{50} = \langle S_z \rangle$$